1 Sinusoids

$\omega = 2\pi f \qquad f = T^{-1} \qquad \theta = \omega t + \phi$
$z = x + \jmath y = Ae^{\jmath\theta} = A\cos\theta + \jmath A\sin\theta = A\angle\phi^\circ$
$x + \jmath y$: $x = A\cos\theta \quad y = A\sin\theta \quad \theta = \tan^{-1}\frac{y}{x} \quad A = \sqrt{x^2+y^2}$
$\frac{A_1\angle\phi_1^\circ}{A_2\angle\phi_2^\circ} = \frac{A_1}{A_2}\angle(\phi_1-\phi_2)^\circ \quad A_1\angle\phi_1^\circ \cdot A_2\angle\phi_2^\circ = A_1\cdot A_2\angle(\phi_1+\phi_2)^\circ$
$A\cos(\omega t) = A\sin\left(\omega t + \frac{\pi}{2}\right) \qquad A\sin(\omega t) = A\cos\left(\omega t - \frac{\pi}{2}\right)$
$y_{\max} = A \qquad y_{\text{rms}} = \frac{y_{\max}}{\sqrt{2}} \qquad y_{\text{rms}} = \sqrt{\frac{1}{T}\int_0^T [y(t)]^2\,dt}$

2 Circuit Basics and Definitions

$V = RI \qquad \mathrm{P} = VI = RI^2 = \frac{V^2}{R} \qquad E(t) = \int \mathrm{p}(t)\,dt$
$i = \frac{\mathrm{d}q}{\mathrm{d}t} \qquad \sum I_{\text{node}} = 0 \qquad V = \frac{\mathrm{d}E_{\mathrm{p}}}{\mathrm{d}q} \qquad \sum V_{\text{loop}} = 0$
$\rho = R\frac{A}{\ell}\ [\Omega\,\mathrm{m}] \qquad \sigma = \frac{\ell}{RA}\ [\mathrm{S\,m^{-1}}] \qquad J = \frac{I}{A}$

2.1 Resistance

$R_{\text{series}} = \sum_i R_i \qquad R_{\text{parallel}} = \left[\sum_i (R_i^{-1})\right]^{-1} \qquad R_{\text{parallel}} = \frac{R_1R_2}{R_1+R_2}$
$V_0, R_{\text{series}} : V_i = \frac{R_i}{R_{\text{series}}}V_0 \qquad I_0, R_{\text{parallel}} : I_i = \frac{R_{\text{parallel}}}{R_i}I_0$
$i_R(t) = \frac{V_{\max}}{R}\sin(\omega t + \phi) \qquad v_R(t) = V_{\max}\sin(\omega t+\phi)$
$V_{\max}e^{\jmath\phi} = RI_{\max}e^{\jmath\phi} \qquad \bar{V} = RI_{\max}\angle\phi^\circ$

2.2 Capacitance

$C = \frac{q}{V} \qquad E_C(t) = \frac{1}{2}Cv_C(t)^2 \qquad C_{\text{plates}} = \varepsilon_0\varepsilon_{\mathrm{R}}\frac{A}{\delta}$
$C_{\text{parallel}} = \sum_i C_i \qquad C_{\text{series}} = \left[\sum_i (C_i^{-1})\right]^{-1}$
$V_{\max} = X_C I_{\max} \qquad X_C = \frac{1}{\omega C} \qquad C = \frac{1}{\omega f_{\mathrm{Q}} R} \qquad C = \frac{I_C^2}{\omega \mathrm{Q}_C}$
$i_C = C\frac{\mathrm{d}v_C}{\mathrm{d}t} \qquad v_C(t) = v_{C0} + \frac{1}{C}\int_0^t i_C(t')\,dt'$
$I_{\max}e^{\jmath\phi_I} = \jmath\omega C V_{\max}e^{\jmath\phi_V} \qquad \bar{I} = \omega C[V_{\max}\angle(\phi_V+90)^\circ]$

2.3 Inductance

$L = \frac{\Phi_B}{I} \qquad E_L(t) = \frac{1}{2}Li_L(t)^2 \qquad L_{\text{coil}} = \frac{N_{\mathrm{t}}\Phi_B}{I} = \frac{\lambda}{I} \approx \frac{\mu_0 N_{\mathrm{t}}^2 A_\sigma}{\ell}$
$L_{\text{series}} = \sum_i L_i \qquad L_{\text{parallel}} = \left[\sum_i (L_i^{-1})\right]^{-1}$
$V_{\max} = X_L I_{\max} \qquad X_L = \omega L \qquad L = \frac{f_{\mathrm{Q}}R}{\omega} \qquad L = \frac{\mathrm{Q}_L}{I_L^2\omega}$
$v_L(t) = L\frac{\mathrm{d}i_L}{\mathrm{d}t} \qquad i_L(t) = i_{L0} + \frac{1}{L}\int_o^t v_L(t')\,dt'$
$V_{\max}e^{\jmath\phi_V} = \jmath\omega L I_{\max}e^{\jmath\phi_I} \qquad \bar{V} = \omega L[I_{\max}\angle(\phi_I+90)^\circ]$

2.4 Impedance and Reactance

$Z = R + \jmath X \qquad |Z| = \sqrt{R^2+X^2} \qquad X = X_L - X_C$
$Z_{\text{series}} = \sum_i Z_i \qquad Z_{\text{parallel}} = \left[\sum_i (Z_i^{-1})\right]^{-1}$
$Z\angle\phi^\circ :\ \phi = \tan^{-1}\frac{X}{R} \qquad R = |\bar{Z}|\cos\phi \qquad X = |\bar{Z}|\sin\phi$
$\bar{Z}_R = R\angle 0^\circ = R$
$\bar{Z}_C = \frac{1}{\omega C}\angle{-90^\circ} = X_C\angle{-90^\circ} \qquad Z_C = \frac{1}{\jmath\omega C} \qquad X_C = \frac{1}{\omega C}$
$\bar{Z}_L = \omega L\angle{+90^\circ} = X_L\angle{+90^\circ} \qquad Z_L = \jmath\omega L \qquad X_L = \omega L$

2.5 Admittance, Conductance, and Susceptance

$Y = G + \jmath B = \frac{1}{Z} = \frac{I}{V} \quad Y_{\text{parallel}} = \sum_i Y_i \quad Y_{\text{series}} = \left[\sum_i (Y_i^{-1})\right]^{-1}$
$G = R^{-1} \qquad B = \frac{1}{X} \qquad B_C = \omega C \qquad B_L = -\frac{1}{\omega L}$

2.5.1 Y Bus Matrix

$$\begin{bmatrix} I_1 \\ I_2 \\ \vdots \\ I_n \end{bmatrix} = \begin{bmatrix} y_{11} & y_{12} & \cdots & y_{1n} \\ y_{21} & y_{22} & \cdots & y_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ y_{n1} & y_{n2} & \cdots & y_{nn} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ \vdots \\ V_n \end{bmatrix}$$

$y_{ii} = \sum_n Y_n$ (into node) $\quad y_{ij} = \sum_n -Y_n$ (directly between nodes)

3 Power

3.1 Average Power

$\mathrm{p} = vi = \frac{1}{2}V_{\max}I_{\max}[\cos(\phi_V - \phi_I) + \cos(2\omega t + \phi_V + \phi_I)]$
$\langle \mathrm{p}\rangle = \frac{1}{T}\int_0^T \mathrm{p}(t)\,dt = \frac{1}{2}V_{\max}I_{\max}\cos(\phi_V-\phi_I) = \frac{RI_{\max}^2}{2} = \frac{V_{\max}^2}{2R}$

3.2 Complex Power, Single Phase

$\bar{V}_{\text{rms}} = \bar{Z}\bar{I}_{\text{rms}} \qquad \phi = \phi_V - \phi_I \qquad \theta_{\text{pf}} = \phi$
$\mathrm{S} = \frac{1}{2}V_{\max}I_{\max}^* = \bar{V}_{\text{rms}}\bar{I}_{\text{rms}}^* = |\bar{I}_{\text{rms}}|^2\bar{Z} = \frac{|\bar{V}_{\text{rms}}|^2}{\bar{Z}^*}$
$\mathrm{S} = \mathrm{P} \pm \jmath\mathrm{Q} \qquad \mathrm{P} = \Re(\mathrm{S}) \qquad \mathrm{Q} = \Im(\mathrm{S}) \qquad |\mathrm{S}| = \sqrt{\mathrm{P}^2+\mathrm{Q}^2}$
$\mathrm{P} = \bar{V}_{\text{rms}}\bar{I}_{\text{rms}}\cos\phi = |\bar{I}_{\text{rms}}|^2R = \frac{|\bar{V}_{\text{rms}}|^2}{|\bar{Z}|^2}R = |\mathrm{S}|\cos\phi = \frac{|\bar{V}_{\text{rms}}|^2\cos\phi}{|\bar{Z}|}$
$\mathrm{Q} = \bar{V}_{\text{rms}}\bar{I}_{\text{rms}}\sin\phi = |\bar{I}_{\text{rms}}|^2\bar{X} = \frac{|\bar{V}_{\text{rms}}|^2}{|\bar{Z}|^2}\bar{X} = |\mathrm{S}|\sin\phi = \frac{|\bar{V}_{\text{rms}}|^2\sin\phi}{|\bar{Z}|}$
$\mathrm{pf} \equiv \frac{\mathrm{P}}{|V_{\text{rms}}||I_{\text{rms}}|} = \frac{\mathrm{P}}{|\mathrm{S}|} = \cos\phi = R/\sqrt{R^2+X^2}$
$\{\theta_{\text{pf}} > 0\}$: (ind, lag, $+\mathrm{Q}$) $\qquad \{\theta_{\text{pf}} < 0\}$: (cap, lead, $-\mathrm{Q}$)
Purely resistive: $\mathrm{P} = \frac{|V|^2}{R}$ $\qquad$ Purely reactive: $\mathrm{Q} = \frac{|V|^2}{X}$
pf correction: $C = \frac{\mathrm{Q}_{\phi 2} - \mathrm{Q}_{\phi 1}}{-|V_{\phi,\text{rms}}|^2\omega} \qquad L = \frac{-|V_{\phi,\text{rms}}|^2}{\omega(\mathrm{Q}_{\phi 2} - \mathrm{Q}_{\phi 1})}$
$\{\bar{Z}_{\text{Ld}} = \bar{Z}_{\mathrm{S}}^*\} \Leftrightarrow \{\mathrm{P} = \mathrm{P}_{\max}\}$

3.3 Complex Power, 3 Phase

Notation: $V_{\alpha\beta} : \alpha > \beta \quad I_{\alpha\beta} : \alpha \to \beta \quad X$: source $\quad x$: load
$\mathrm{S_T} = \sum \mathrm{S}_\phi = |\bar{V}_{\text{Ln}}||\bar{I}_{\text{Ln}}|\cdot\sqrt{3}\angle\phi_{\bar{Z}}^\circ$
$|\bar{I}_{\text{Ln}}| = |\bar{I}_{an}| = |\bar{I}_{nb}| = |\bar{I}_{nc}|$
$|\bar{V}_{\text{Ln}}| = |\bar{V}_{ab}| = |\bar{V}_{bc}| = |\bar{V}_{ca}|$
$\bar{Z}_\Delta = 3\bar{Z}_Y \qquad \Delta\phi = 120^\circ$
$\bar{V}_{\Delta,AB} = \bar{V}_{Y,AN}\cdot\sqrt{3}\angle 30^\circ$

3.3.1 Y Source / Y Load

$\bar{V}_{\text{Ln}} = \bar{V}_\phi \cdot \sqrt{3}\angle 30^\circ$
$\bar{V}_{AB} = \bar{V}_{AN} - \bar{V}_{BN} = \bar{V}_{AN}\cdot\sqrt{3}\angle 30^\circ \qquad \sum_i \bar{V}_{iN} = 0$
Y-Y: $\bar{I}_{an} = \frac{\bar{V}_{AN}}{\bar{Z}_Y + \bar{Z}_{\text{Ln}}} \qquad \mathrm{S}_{\phi a} = \bar{V}_{an}\bar{I}_{an}^* \qquad \bar{V}_{an} = \bar{I}_{\text{Ln}}\bar{Z}_Y$

3.3.2 Δ Source / Δ Load

$\bar{I}_{\text{Ln}} = \bar{I}_\phi\cdot\sqrt{3}\angle{-30^\circ}$
$\bar{I}_{Aa} = \bar{I}_{ab} - \bar{I}_{ca} = \bar{I}_{ab}\cdot\sqrt{3}\angle{-30^\circ} \qquad \sum \bar{I}_{\text{node}} = 0$
Δ-Δ: $\bar{I}_{ab} = \frac{\bar{V}_{AB}}{\bar{Z}_\Delta} \qquad \mathrm{S}_{\phi a} = \bar{V}_{AB}\bar{I}_{ab}^*$

3.3.3 Y Source / Δ Load

$\bar{V}_{ab} = \bar{V}_{AB} = \bar{V}_{AN}\cdot\sqrt{3}\angle 30^\circ$
$\bar{I}_{ab} = \frac{\bar{V}_{AB}}{\bar{Z}_\Delta} \qquad \bar{I}_{Aa} = \bar{I}_{ab}\cdot\sqrt{3}\angle{-30^\circ}$
$\mathrm{S}_{\phi a} = \bar{V}_{AB}\bar{I}_{ab}^* = (\bar{V}_{AN}\cdot\sqrt{3}\angle 30^\circ)\bar{I}_{ab}^*$

4 Magnetically Coupled Circuits

$\vec{F}_{\text{surfaces}} = \frac{\vec{B}^2 A}{2\mu_0} \qquad \mathscr{F} = N_{\mathrm{t}}I = \Phi_B\mathscr{R} \qquad |\Phi| \propto |I|$
$\mathscr{R} = \frac{\ell}{\mu A} \qquad \mu = \mu_0\mu_{\mathrm{R}} \qquad \vec{B} = \frac{\Phi_B}{A} = \frac{\mu\vec{\mathscr{F}}}{\ell} = \mu\vec{H}$
$\mu_0 = 4\pi\mathrm{E}{-7}\,\frac{\mathrm{H}}{\mathrm{m}} \qquad \varepsilon_0 = 8.854\mathrm{E}{-12}\,\frac{\mathrm{F}}{\mathrm{m}} \qquad c_0 = (\mu_0\varepsilon_0)^{-\frac{1}{2}}$

4.1 1 Phase Ideal Transformers

$v_i = N_{\mathrm{t},i}\frac{\mathrm{d}\Phi_{Bi}}{\mathrm{d}t} \qquad V_1I_1 = V_2I_2$
$\{\Phi_{B1} = \Phi_{B2}\} : \frac{V_1}{V_2} = \frac{N_{\mathrm{t},1}}{N_{\mathrm{t},2}} \equiv a \qquad \frac{I_1}{I_2} = \frac{N_{\mathrm{t},2}}{N_{\mathrm{t},1}} = \frac{1}{a}$
$V_1 = aV_2 \qquad I_1 = \frac{I_2}{a} \qquad Z_1 = a^2Z_2$
1: source $\quad$ 2: load $\quad \{a > 1\}$: step-down $\quad \{a < 1\}$: step-up
$\Phi_B(t) = A_\Phi\sin\omega t \qquad v = 2\pi f N_{\mathrm{t}}A_\Phi\cos\omega t \qquad A_\Phi \equiv \Phi_{B,\max}$
$V_{\max} = 2\pi f N_{\mathrm{t}}A_\Phi \qquad V_{\text{rms}} = \frac{2\pi}{\sqrt{2}}fN_{\mathrm{t}}A_\Phi$
$\mathrm{S}_{\text{rated}} = V_{\text{rated}}I \qquad A_{\sigma,\min} = \frac{A_\Phi}{\vec{B}_{\text{material}}}$

4.2 1 Phase Real Transformers

$\bar{V}_1 = \bar{a}\bar{V}_2 = |\bar{a}||\bar{V}_2|\angle(\phi_{V2}+\phi_a)^\circ \qquad \bar{I}_1 = \frac{\bar{I}_2}{\bar{a}^*} \qquad \bar{Z}_1 = |\bar{a}|^2\bar{Z}_2$
$\hat{\mathrm{P}}_{\mathrm{e}} = k_{\mathrm{e}}f^2\vec{B}_{\max}^2\delta^2 \qquad \hat{\mathrm{P}}_{\mathrm{h}} = k_{\mathrm{h}}f\vec{B}_{\max}^{n_{\mathrm{S}}}$

4.2.1 SC-OC Test

$Z_{\text{eq1}} = \frac{V_{\text{sc}}}{I_{\text{sc}}} = R_{\text{eq1}} + \jmath X_{\text{eq1}} \qquad R_{\text{eq1}} = \frac{\mathrm{P_{sc}}}{I_{\text{sc}}^2} \qquad X_{\text{eq1}} = \sqrt{Z_{\text{eq1}}^2 - R_{\text{eq1}}^2}$
$\phi = \cos^{-1}\frac{\mathrm{P_{oc}}}{I_{\text{oc}}V_{\text{oc}}} \qquad I_{\mathrm{c}} = I_{\text{oc}}\cos\phi \qquad I_{\mathrm{m}} = I_{\text{oc}}\sin\phi$
$R_{\text{core2}} = \frac{V_{\text{oc}}}{I_{\text{core}}} \qquad X_{\text{mutual2}} = \frac{V_{\text{oc}}}{I_{\text{mutual}}} \qquad R_{\text{eq2}} = \frac{R_{\text{eq1}}}{a^2} \qquad X_{\text{eq2}} = \frac{X_{\text{eq1}}}{a^2}$

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$x=\frac{\mathrm{S_{Ld}}}{\mathrm{S_{rated}}} \quad \eta \equiv \frac{x\cdot \mathrm{S_{rated}}\cdot \mathrm{pf}}{x\cdot \mathrm{S_{rated}}\cdot \mathrm{pf}+\underbrace{\mathrm{P_{oc}}}_{\mathrm{P_{core}}}+x^2\underbrace{\mathrm{P_{sc}}}_{\mathrm{P_{Cu\,FL}}}} \quad \{x^2\mathrm{P_{Cu\,FL}}=\mathrm{P_{core}}\}:\eta_{\max}$

4.2.2 Voltage Regulation

$\frac{V_1}{a}=\bar{V}_{\mathrm{Ld}}+\bar{I}_{\mathrm{Ld}}(R_{\mathrm{eq2}}+\jmath X_{\mathrm{eq2}})$

$V_{\mathrm{R,NL}}=\frac{|V_1|}{a} \qquad V_{\mathrm{R,FL}}=|V_{\mathrm{rated}}| \qquad \mathrm{VR}=\frac{V_{\mathrm{R,NL}}-V_{\mathrm{R,FL}}}{V_{\mathrm{R,FL}}}\times 100\,\%$

$\frac{V_1}{a}=\sqrt{(|V_{\mathrm{Ld}}|\cos\phi+|I_{\mathrm{Ld}}|R_{\mathrm{eq2}})^2+(|V_{\mathrm{Ld}}|\sin\phi\pm|I_{\mathrm{Ld}}|X_{\mathrm{eq2}})^2}$

$(+)$ lagging, unity $\qquad (-)$ leading

5 Per Unit System

$F\in\{V,I,R,Z,X,\mathrm{S}\}:\ \dot{F}\cdot F_{\mathrm{b}}=F_{\mathrm{actual}}$

$\mathrm{S_b}=\bar{V}_{\mathrm{b}}\bar{I}_{\mathrm{b}}^{*} \qquad \bar{Z}_{\mathrm{b}}=\frac{\bar{V}_{\mathrm{b}}^2}{\mathrm{S_b}} \qquad \dot{\mathrm{S}}=\dot{\mathrm{P}}\pm\jmath\dot{\mathrm{Q}}=\dot{\bar{V}}\dot{\bar{I}}^{*}$

$\bar{Z}_{\mathrm{b},i}=\frac{\bar{V}_{\mathrm{b},i}^2}{\mathrm{S_b}} \qquad \bar{I}_{\mathrm{b},i}=\frac{\mathrm{S_b}}{\bar{V}_{\mathrm{b},i}} \qquad \bar{V}_{\mathrm{b},i}=\bar{V}_{\mathrm{b},0}\frac{V_{i,0}}{V_0}$ [HV or LV basis]

5.1 Lossy Transformer

$\eta\equiv\frac{x\cdot\mathrm{pf}}{x\cdot\mathrm{pf}+\frac{\mathrm{P_{core}}}{\mathrm{S_b}}+x^2\dot{R}} \qquad \dot{R}=\dot{\mathrm{P}}_{\mathrm{Cu\,FL}}=\frac{\mathrm{P_{Cu\,FL}}}{\mathrm{S_b}}=\frac{R_{\mathrm{eq1}}}{Z_{\mathrm{b1}}}=\frac{R_{\mathrm{eq2}}}{Z_{\mathrm{b2}}}$

5.2 3 Phase Power

$\bar{V}_{\mathrm{b},AN}=\frac{\bar{V}_{\mathrm{b},AB}}{\sqrt{3}} \qquad |\mathrm{S_T}|=\sqrt{3}|V_{AB}||I_{Aa}|$

$I_{\mathrm{b},3\phi}=\frac{\mathrm{S}_{\mathrm{b},3\phi}}{\sqrt{3}V_{\mathrm{b},3\phi,AB}}=\frac{\mathrm{S}_{\mathrm{b},1\phi}}{V_{\mathrm{b},1\phi}} \qquad Z_{\mathrm{b},3\phi}=\frac{V_{\mathrm{b},3\phi,AB}^2}{\mathrm{S}_{\mathrm{b},3\phi}}$

5.3 Tertiary Transformer Star Equivalent Circuit

1 excited, 2 shorted: $\frac{V}{I}=Z_{12}$ $\quad$ 1 excited, 3 shorted: $\frac{V}{I}=Z_{13}$

2 excited, 3 shorted: $\frac{V}{I}=Z_{23,\,\mathrm{sec.}}$ $\quad Z_{23,\,\mathrm{primary}}=\left(\frac{N_{\mathrm{t},1}}{N_{\mathrm{t},2}}\right)^2 Z_{23,\,\mathrm{sec.}}$

$X_1=\frac{1}{2}(Z_{12}+Z_{13}-Z_{23}) \qquad X_2=\frac{1}{2}(Z_{12}+Z_{23}-Z_{13})$

$X_3=\frac{1}{2}(Z_{13}+Z_{23}-Z_{12})$

$Z_{\mathrm{b},1}=\frac{V_{\mathrm{b},1}}{S_{\mathrm{b}}} \qquad \dot{Z}_1=\frac{\jmath X_1}{Z_{\mathrm{b},1}} \qquad \dot{Z}_2=\frac{\jmath X_2}{Z_{\mathrm{b},1}} \qquad \dot{Z}_3=\frac{\jmath X_3}{Z_{\mathrm{b},1}}$

5.4 Off Nominal Turns Ratio, Pi Equivalent

Transformer nameplate: $\dot{Z}_i'=\dot{Z}_i\left(\frac{V_{\mathrm{b},i}}{V_{\mathrm{b},0}'}\right)^2\frac{\mathrm{S}_{\mathrm{b},0}'}{\mathrm{S}_{\mathrm{b},i}}$ $\quad$ [b, 0 in basis zone]

$Z_{\mathrm{T,\,HV}}=c^2 Z_{\mathrm{T,\,LV}} \qquad c\equiv\sqrt{\frac{\dot{Z}_{\mathrm{T,\,HV}}'}{\dot{Z}_{\mathrm{T,\,LV}}'}} \qquad Y_{\mathrm{eq}}=(\dot{Z}_{\mathrm{T,\,HV}}')^{-1}$

$Y_{10}=(1-c)Y_{\mathrm{eq,HV}} \qquad Y_{12}=cY_{\mathrm{eq}} \qquad Y_{20}=(|c|^2-c)Y_{\mathrm{eq}}$

6 Transmission

$A\,[\mathrm{CM}]=(\varnothing\,[\mathrm{mil}])^2 \quad 1\ \mathrm{mil}=0.001\mathrm{in} \quad 1\ \mathrm{CM}=5.066\times10^{-10}\ \mathrm{m}^2$

$R=\frac{\rho(T)\ell}{A_{\sigma,\mathrm{T}}} \qquad \rho(T_2)=\rho(T_1)\frac{M+T_2}{M+T_1}$

$\bar{\delta}_{m\times n}=\sqrt[m\cdot n]{\Pi_{i=1}^{m}\Pi_{j=1}^{n}\delta(m_i,n_j)}$

$\bar{R}_L=\sqrt[n^2]{\Pi_{i=1}^{n}\Pi_{j=1}^{n}\delta_{ij}} \qquad \delta_{i=j}\equiv e^{-0.25}R_{\mathrm{wire}}$

$\bar{R}_C=\sqrt[n^2]{\Pi_{i=1}^{n}\Pi_{j=1}^{n}\delta_{ij}} \qquad \delta_{i=j}\equiv R_{\mathrm{wire}}$

$\tilde{X}_L=\omega\tilde{L} \qquad \tilde{Y}=\omega\tilde{C} \qquad \tilde{Z}=\tilde{R}+\jmath\tilde{X}$

6.1 Inductance

$\tilde{\lambda}_{\mathrm{int}}=\frac{\mu_0 I}{8\pi} \quad \tilde{L}_{\mathrm{int}}=\frac{\mu_0}{8\pi} \qquad \tilde{\lambda}_{\mathrm{ext}}=\frac{\mu I}{2\pi}\ln\frac{\bar{\delta}}{R_{\mathrm{wire}}} \quad \tilde{L}_{\mathrm{ext}}=\frac{\mu_0}{2\pi}\ln\frac{\delta}{R_{\mathrm{wire}}}$

$\tilde{L}_{\mathrm{line}}=2\mathrm{E}{-}7\,\frac{\mathrm{H}}{\mathrm{m}}\ln\frac{\bar{\delta}}{\bar{R}_L}$

Single phase, 2 solid conductors:

$\tilde{L}_{\mathrm{loop}}=L_1+L_2=4\mathrm{E}{-}7\,\frac{\mathrm{H}}{\mathrm{m}}\ln\frac{\delta(12)}{\sqrt[2]{e^{-0.5}R_1R_2}}$

3 phase, symmetrically spaced solid conductors:

$\tilde{L}_\phi=\frac{\tilde{\lambda}_\phi}{I_\phi} \qquad \tilde{\lambda}_\phi=2\mathrm{E}{-}7\,\frac{\mathrm{H}}{\mathrm{m}}I_\phi\ln\frac{\delta(12)}{\bar{R}_L} \qquad \bar{R}_L=e^{-0.25}R_{\mathrm{wire}}$

Bundled conductors: $\bar{R}_L=\sqrt[n]{\Pi_{i=1}^{m}\delta_{1i}}$

3ϕ transpose: $\tilde{L}_\phi=2\mathrm{E}{-}7\,\frac{\mathrm{H}}{\mathrm{m}}\ln\frac{\sqrt[3]{\delta(12)\delta(23)\delta(31)}}{\bar{R}_L}$

6.2 Capacitance

Single phase, 2 solid conductors:

$C_{AB}=\frac{q}{V_{AB}} \qquad C_{AN}=\frac{q}{\frac{1}{2}V_{AB}}$

$\tilde{C}_{AN}=2\pi\varepsilon_0\left(\ln\frac{\bar{\delta}}{\bar{R}_C}\right)^{-1}$

3ϕ transpose: $\tilde{C}_{AN}=2\pi\varepsilon_0\left(\ln\frac{\sqrt[3]{\delta(12)\delta(23)\delta(31)}}{\bar{R}_C}\right)^{-1}$

$\tilde{C}_{A\text{♁}}=2\pi\varepsilon_0\left(\ln\frac{\bar{\delta}}{\bar{R}_C}-\ln\frac{\sqrt[3]{\delta(AB')\delta(BC')\delta(CA')}}{\sqrt[3]{\delta(AA')\delta(BB')\delta(CC')}}\right)^{-1}$

$I_{\mathrm{charging}}=YV_{AN}=\jmath2\pi fC_{AN}V_{AN}$

$\mathrm{Q}_{\mathrm{charging},\phi}=YV_{AN}^2=\omega C_{AN}V_{AN}^2$

$\mathrm{Q}_{\mathrm{charging,T}}=3YV_{AN}^2=3\omega C_{AN}V_{AN}^2=\omega C_{AN}V_{AB}^2$

6.3 Distributed Line

$\begin{bmatrix}\dot{V}_{\mathrm{S}}\\ \dot{I}_{\mathrm{S}}\end{bmatrix}=\underbrace{\begin{bmatrix}A & B\\ C & D\end{bmatrix}}_{\boldsymbol{T}}\begin{bmatrix}\dot{V}_{\mathrm{R}}\\ \dot{I}_{\mathrm{R}}\end{bmatrix} \qquad \begin{matrix}\dot{V}_{\mathrm{S}}=A\dot{V}_{\mathrm{R}}+B\dot{I}_{\mathrm{R}}\\ \dot{I}_{\mathrm{S}}=C\dot{V}_{\mathrm{R}}+D\dot{I}_{\mathrm{R}}\end{matrix}$

$\gamma\equiv\alpha+\jmath\beta=\sqrt{ZY}\,[\mathrm{m}^{-1}] \qquad Z_0=\sqrt{\frac{Z}{Y}}\,[\Omega] \qquad A,D[=]\mathrm{pu}$

6.3.1 Exact Pi (>250 km)

$\boldsymbol{T}=\begin{bmatrix}\cosh\gamma\ell & Z_0\sinh\gamma\ell\\ \frac{1}{Z_0}\sinh\gamma\ell & \cosh\gamma\ell\end{bmatrix} \qquad \frac{Y_{\mathrm{E}}}{2}=\frac{1}{Z_0}\tanh\frac{\gamma\ell}{2}$

6.3.2 Nominal Pi (80 km to 250 km)

$\boldsymbol{T}=\begin{bmatrix}1+\frac{Z_{\mathrm{N}}Y_{\mathrm{N}}}{2} & Z_{\mathrm{N}}\\ Y_{\mathrm{N}}\left(1+\frac{Z_{\mathrm{N}}Y_{\mathrm{N}}}{4}\right) & 1+\frac{Z_{\mathrm{N}}Y_{\mathrm{N}}}{2}\end{bmatrix} \qquad \begin{matrix}Z_{\mathrm{N}}=\ell Z\\ \frac{Y_{\mathrm{N}}}{2}=\ell\frac{Y}{2}\end{matrix}$

6.3.3 Short Line (<80 km)

$\boldsymbol{T}=\begin{bmatrix}1 & Z_{\mathrm{S}}\\ 0 & 1\end{bmatrix} \qquad \begin{matrix}Z_{\mathrm{S}}=\ell Z\\ \frac{Y_{\mathrm{S}}}{2}=0\end{matrix}$

6.3.4 Lossless Line

$Z_0=\sqrt{\frac{L}{C}}\Omega \qquad \gamma=\jmath\omega\sqrt{LC}\mathrm{m}^{-1} \qquad \lambda=\frac{1}{f\sqrt{LC}}$

$\boldsymbol{T}(x)=\begin{bmatrix}\cosh\gamma x & \jmath Z_0\sinh\gamma x\\ \frac{1}{Z_0}\sinh\gamma x & \cosh\gamma x\end{bmatrix} \qquad \begin{matrix}Z_{\mathrm{L}}=\jmath X_{\mathrm{L}}\\ \frac{Y_{\mathrm{L}}}{2}=\ell\frac{\jmath\omega C_{\mathrm{L}}}{2}\end{matrix}$

$\mathrm{SIL}:I_{\mathrm{R}}=\frac{|V|_{\mathrm{R}}}{Z_0} \qquad \mathrm{S}(x)=V(x)I^*(x)=\frac{|V_{\mathrm{R}}|^2}{Z_0}=\frac{V_{\mathrm{rated}}^2}{Z_0}$

6.3.5 Steady State Stability Limit

$\mathrm{P_R}=\frac{|V_{\mathrm{R}}||V_{\mathrm{S}}|}{|X_{\mathrm{L}}|}\sin(\phi_{V,\mathrm{S}}-\phi_{V,\mathrm{R}})=\dot{V}_{\mathrm{R}}\dot{V}_{\mathrm{S}}\cdot\mathrm{SIL}\cdot\frac{\sin(\phi_{V,\mathrm{S}}-\phi_{V,\mathrm{R}})}{\sin\left(\frac{2\pi\ell}{\lambda}\right)}$

6.3.6 Maximum Power Flow

$|A|\angle\phi_A{}^\circ\equiv\boldsymbol{T}(A) \qquad |B|\angle\phi_B{}^\circ\equiv\boldsymbol{T}(B)$

$\mathrm{S_R}=\frac{|V_{\mathrm{R}}||V_{\mathrm{S}}|}{|B|}\angle(\phi_B-\phi_{V,\mathrm{S}}-\phi_{V,\mathrm{R}})^\circ+\frac{|V_{\mathrm{R}}|^2|A|}{|B|}\angle(\phi_B-\phi_A-180)^\circ$

References

`ISBN-13: 9781111425791`